

A Cutoff Constrained by the Oblique Parameters in Electroweak Theory with Two Massless Higgs Doublets

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Abstract

Electroweak theory with two massless Higgs doublets is studied by solving renormalization group equations for coupling constants in one-loop approximation. A cutoff Λ , at which one of quartic couplings in the Higgs potential blows up, is obtained by imposing constraints from the oblique parameter T on the quartic couplings at low energy. We find $\Lambda \simeq 0.52 \sim 8.4$ TeV at the Higgs mass $M_H = 100$ GeV. The cutoff Λ is at most about 60 TeV even if we take into account the LEP lower bound of $M_H \simeq 64$ GeV. It cannot reach the Planck or GUT scale due to severe experimental constraints. It is impossible in the model to realize a large gauge hierarchy as suggested many years ago by S. Weinberg.

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1 Introduction

It was pointed out by S. Weinberg many years ago[1] that massless-Higgs-doublet models, in which radiative corrections induce the spontaneous breakdown of the $SU(2) \times U(1)$ gauge symmetry (Coleman-Weinberg mechanism[2]), may have a possibility to explain the gauge hierarchy. Even though the quartic couplings of scalar fields defined at high energy scale, say the Planck (or GUT) scale $M_{P(G)}$, are positive for the stability of theories, they decrease due to the radiative effects of gauge and Yukawa couplings as energy scale goes down. New minimum occurs in the potential aside from the origin by radiative corrections at a certain low-energy scale, which gives the same order of the VEVs for scalar fields. An enormous small mass ratio of the weak scale M_W and $M_{P(G)}$ arises as an immediate dynamical consequence of massless scalar theories coupled to gauge fields. Yukawa couplings, in general, tend to destabilize the new minimum. The large Yukawa coupling of $O(1)$ makes the new minimum unstable in one-massless-Higgs-doublet model, so that the model is excluded because of the recently announced heavy top quark mass $M_t \simeq 175$ GeV. We believe that it is worthwhile to study this attractive possibility for realizing the gauge hierarchy even in models beyond the minimal model.

Now the electroweak measurements are so precise that we are at the stage that we can say something about new physics beyond the minimal standard model. The oblique parameters introduced by Peskin and Takeuchi[3] are very useful and transparent tools for probing or constraining new physics. In a previous paper[4] we studied the oblique parameters in electroweak theory with two massless Higgs doublets. There are four kinds of scalars in the model, charged Higgs H^\pm , CP even (odd) Higgs h (A) and scalon H . The scalon is identified with the usual Higgs scalar in the minimal standard model and its mass is fixed to define the experimental limits on the oblique parameters. Hereafter the above Higgs masses are denoted as M_{H^\pm} , M_h , M_A and M_H , respectively. We obtained an allowed region of masses of new particles by studying the oblique parameter T . A mass relation induced by the Coleman-Weinberg mechanism played an important and essential role for obtaining it. The current experiments strictly constrain the mass spectra of new particles in

the model. This means that the quartic couplings in the Higgs potential, which are written in terms of the masses of new particles, are also well-constrained.

We think that what Weinberg suggested is very attractive for realizing the gauge hierarchy even though we have at present no theoretically reliable mechanism to guarantee the masslessness of the Higgs doublets at high energy scale. It is important to study the possibility of such a large gauge hierarchy in the two-massless-Higgs-doublets model, taking into account comprehensively the constraints from the oblique parameters on coupling constants, in particular, on the quartic couplings in the Higgs potential and the heavy top quark mass. It is also interesting to consider what physical pictures we should make if we cannot get a large gauge hierarchy.

In this paper we solve the RGE's for coupling constants in the electroweak theory with two massless Higgs doublets in one-loop approximation¹ to obtain a cutoff Λ in the model at which one of the quartic couplings blows up. We ignore the effects of the Yukawa couplings except for that of the top quark in evaluating the RGE's. Initial conditions for quartic couplings in the equations are determined from the constraints obtained by the oblique parameter T at reference points $(M_t, M_H) = (175, 100), (150, 100), (125, 100)$ GeV, where M_t is the top quark mass and M_H is the mass of the scalon. At each reference point, we obtain allowed mass regions for new particles. Picking up some points in the allowed region and fixing $\tan\beta$ for each point, which is a ratio of the vacuum expectation values of two Higgs fields, one can completely fix the quartic couplings at low energy. In our numerical analyses we choose $\tan\beta = 0.6, 1.0, 3.0$. Using the quartic couplings obtained in this way as the initial conditions at low energy, we obtain a cutoff Λ in the model. We find that (i) the cutoff Λ is suppressed if there are hierarchies among the quartic couplings, while degeneracies between all the quartic couplings give larger Λ (ii) Λ 's do not take quite different values among these reference points, and it is about $\Lambda \simeq 0.52 \sim 8.4$ TeV. (iii) If M_H becomes smaller it is possible to make Λ larger than these values. It is, however, at most about 60 TeV if we take into account the current experimental limits on the top quark and Higgs masses.

¹ β functions for various couplings in the model are derived in, for example, [5][6]

In the next section, we briefly review the Higgs sector of the electroweak theory with two massless Higgs doublets. We clarify the relations between masses of new particles and the quartic couplings in the Higgs potential. In section 3 we show the allowed regions of masses of new particles constrained from the experimental limit on the oblique parameter T at three different reference points. In the section 4 we present results of our numerical analyses. Concluding remarks are given the final section.

2 The Higgs sector of the model

The Higgs potential of our model is given by

$$\begin{aligned} V_H = & \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}]. \end{aligned} \quad (2.1)$$

For the tree-level potential to be stable, the parameters $\lambda_{1\sim 5}$ must satisfy

$$\lambda_{1,2} > 0, \quad \lambda_5 < 0, \quad \lambda_4 + \lambda_5 < 0, \quad (2.2)$$

$$\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 + \lambda_5 \geq 0. \quad (2.3)$$

Here we choose the sign of λ_5 to be negative. The third condition in (2.2) must be satisfied to keep the $U(1)_{\text{em}}$ invariance. The Higgs potential is a homogeneous polynomial of the Higgs fields so that the $SU(2) \times U(1)$ gauge symmetry is not broken at the tree level. One must take into account at least one-loop corrections to the potential in order to break the symmetry. Detailed discussions on the one-loop effective potential in the model are given in [5][7][8]. It is important to note that the gauge symmetry is broken desirably by radiative corrections (Coleman-Weinberg mechanism) if and only if the tree-level potential possesses a flat direction realized by the coupling relation

$$\lambda(M_R) \equiv \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 + \lambda_5 = 0 \quad (2.4)$$

at some renormalization scale M_R . The flat direction in the VEV space of the Higgs potential is given by

$$\begin{aligned} \langle \Phi_1 \rangle = & \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} = \frac{\rho}{\sqrt{2}} \begin{pmatrix} 0 \\ n_{01} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} = \frac{\rho}{\sqrt{2}} \begin{pmatrix} 0 \\ n_{02} \end{pmatrix}, \\ (\rho \geq 0), \end{aligned}$$

where

$$n_{01}^2 = \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1} + \sqrt{\lambda_2}}, \quad n_{02}^2 = \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_1} + \sqrt{\lambda_2}}.$$

The order parameter ρ^2 is fixed by

$$\rho^2 = v^2 \equiv \frac{1}{\sqrt{2}G_F},$$

where G_F is the Fermi coupling constant. The renormalization scale M_R at which (2.4) is realized is related to v by

$$M_R^2 = e^{-11/3} v^2$$

through the Coleman-Weinberg mechanism.

The physical Higgs masses are obtained by the quadratic terms of the potential with respect to fields. We have

$$M_{H^\pm}^2 = \frac{1}{2}(\sqrt{\lambda_1\lambda_2} + \lambda_3)v^2, \quad M_A^2 = -\lambda_5v^2, \quad M_h^2 = \sqrt{\lambda_1\lambda_2}v^2, \quad (2.5)$$

$$M_H^2 = \frac{G_F}{4\sqrt{2}\pi^2} [6M_W^4 + 3M_Z^4 - 12M_t^4 + 2M_{H^\pm}^4 + M_h^4 + M_A^4]. \quad (2.6)$$

We identify the scalar denoted by H , which is the pseudo-Goldstone boson associated with the scale invariance of the Higgs potential, as the usual Higgs boson in the minimal standard model. Its mass is defined by $M_H^2 \equiv d^2V_1/d\rho^2|_{\rho=v}$, where V_1 is the one-loop effective potential in our model. By defining $\tan\beta$ as

$$\tan^2\beta \equiv \frac{v_2^2}{v_1^2} = \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}}, \quad (2.7)$$

the five quartic couplings are written in terms of the masses of new particles;

$$\lambda_1 = \frac{M_h^2}{v^2} \tan^2\beta, \quad \lambda_2 = \frac{M_h^2}{v^2} \frac{1}{\tan^2\beta}, \quad (2.8)$$

$$\lambda_3 = \frac{1}{v^2}(2M_{H^\pm}^2 - M_h^2), \quad \lambda_4 = \frac{1}{v^2}(M_A^2 - 2M_{H^\pm}^2), \quad \lambda_5 = -\frac{M_A^2}{v^2}. \quad (2.9)$$

From (2.8) and (2.9) we see that $\lambda_{3\sim 5}$ are fixed once we fix the masses M_i ($i = H^\pm, h, A$). The determination of $\tan\beta$ fix λ_1 and λ_2 . Note that all the quartic couplings are defined at the energy scale M_R . The masses of new particles are constrained by the oblique parameter T as we will see in the next section.

3 Constraints from the oblique parameter T

Let us obtain the allowed mass region for new particles, M_{H^\pm} , M_A and M_h by studying the oblique parameters. The contributions to the parameters from new particles are studied in [4] at a reference point I(M_t, M_H) = (175, 100) GeV in which we found that the constraint on the masses mainly comes from the parameter T . We had almost no constraints on them from the parameter S at the reference point. This situation does not change in the case for reference points II and III defined below.

In order to see the behavior of the cutoff in our model with respect to the heaviness of the top quark, we choose following reference points;

$$(M_t, M_H) = \text{I}(175, 100), \text{ II}(150, 100), \text{ III}(125, 100) \text{ GeV}$$

At each reference point, the experimental limits on T are calculated to be[10][11]

$$\begin{aligned} -0.62 &\leq T \leq -0.14 \text{ for I} \\ -0.32 &\leq T \leq +0.16 \text{ for II} \\ -0.06 &\leq T \leq +0.42 \text{ for III.} \end{aligned} \quad (3.1)$$

In Figs. 1 ∼ 3 we display the allowed mass region for M_{H^\pm} and M_h by using (3.1). The areas inside the solid curves are the allowed mass regions from the 1σ errors in T . In the previous paper we assumed $M_h \leq M_A$ because the parameter T is symmetric under $M_h \leftrightarrow M_A$. In this paper, however, we do not assume any possible mass hierarchies among new particles to study allowed quartic couplings comprehensively. At the reference point I, T takes negative values at the 1σ errors. A relation $M_h < M_{H^\pm} < M_A$ or $M_A < M_{H^\pm} < M_h$ must be satisfied to obtain the negative T . A left (right) domain of the allowed regions in Fig. 1 corresponds to the case $M_{h(A)} < M_{H^\pm} < M_{A(h)}$. In order for the parameter T to make sense, the masses of new particles must be larger than the Z -boson mass M_Z . We set M_i ($i = H^\pm, h, A$) ≥ 140 GeV for illustration. This limit does not alter our numerical results for the cutoff significantly. Note that the values of M_{H^\pm} can be determined once we fix M_A and M_h . This is because, from the expression (2.6), one obtains the mass

relation

$$M_{H^\pm}^2 = \left[\frac{2\sqrt{2}\pi^2}{G_F} M_H^2 - \frac{1}{2}(6M_W^4 + 3M_Z^4 - 12M_t^4) - \frac{1}{2}(M_A^4 + M_h^4) \right]^{1/2}. \quad (3.2)$$

The oblique parameter T depends on three parameters, M_{H^\pm} , M_h and M_A , but the mass relation (3.2) reduces the number of free parameters to two, for example, M_A and M_h .

For the later analyses we note the custodial symmetry of the Higgs potential in our model. The experimental limits on T include the value of zero in cases of the reference points II and III. In order to explain this value the Higgs potential must have the custodial symmetry. The custodial symmetry in our model can be realized if $M_{H^\pm} = M_A$ or $M_{H^\pm} = M_h$ [9]. The former case implies $\lambda_4 = \lambda_5$ with $\tan\beta$ being free, and the later case does $\lambda_1 = \lambda_2 = \lambda_3$ with $\tan\beta = 1.0$. The custodial symmetry is not necessary in the case of the reference point I because T is negative at the 1σ errors. We also note that if all the masses of new particles are degenerate we have $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_4 = -\lambda_5$ for $\tan\beta = 1.0$.

4 Numerical results

Let us pick up some points which cover the almost all allowed region of masses of new particles obtained in the section 3. The points we pick up are labeled by A, B, *etc* in Figs. 1 ~ 3. For the right domain of the allowed regions in Fig. 1, we take those points which are obtained from the points in the left domain by an exchange of M_h and M_A . This is possible because both domains are related each other by this exchange due to the symmetric property of T and M_{H^\pm} under $M_h \leftrightarrow M_A$. At each point we calculate the quartic couplings $\lambda_{1\sim 5}$ by (2.8) and (2.9) for each $\tan\beta = 0.6, 1.0, 3.0$. We use these couplings as initial conditions of the RGE's at low energy in our model. One should be careful that the conditions (2.2) and (2.3) must be satisfied in evaluating the RGE's, otherwise the system we concern becomes unstable. The absolute values of the initial conditions we shall use are within ranges $0.05 < |\lambda_i| < 4.0$ ($i = 1 \sim 5$). The quartic couplings constrained by T are large enough, so that the heavy top quark such as $M_t \sim 175$ GeV

does not destabilize the true minimum of the potential in our model. This point is quite different from the model with one massless Higgs doublet.

In Tables 1 ∼ 3 we present numerical results of the values of a cutoff Λ for the reference points I ∼ III, respectively ². There are the cases with $\Lambda < M_i$, where i stands for H^\pm, h or A . These cases denoted by values in parentheses in the tables are not acceptable. We observe that Λ 's do not take quite different values among these tables. This means that the effect of the heavy top quark on Λ does not appear seriously at these reference points. This can be traced back to magnitudes of the mass of the scalon we choose in the reference points. It is seen from (2.6) that the quartic couplings must be appropriately large enough in order for the mass of the scalon to be 100 GeV for the heavy top quark masses. The initial values of the quartic couplings are sufficiently large, so that the running of the couplings is not affected strongly by the large Yukawa coupling, that is, the heavy top quark. Therefore, the sensitivity of the cutoff to the heavy top quark is small and the values of Λ is also small at the reference points. We will discuss the possibilities for obtaining larger cutoff in the last paragraph of this section.

At each point in the tables the largest cutoff is obtained for $\tan\beta = 1.0$, which means $\lambda_1 = \lambda_2$. A large difference $\lambda_1 \ll (\gg) \lambda_2$ are produced when $\tan\beta = 0.6$ (3.0) (see (2.7)). Then, one of the couplings blows up faster than the other coupling. This is why there are many $\Lambda < M_i$ cases in these two values of $\tan\beta$. When λ_1 or λ_2 exceeds about 5, the cutoff Λ lies below one of mass spectra of new particles as far as our numerical analyses are concerned. For example, at the point E (M) with $\tan\beta = 3.0$ (0.6) in Table 2, we have $(\lambda_1, \lambda_2) \simeq (5.9 (0.74), 0.07 (5.7))$.

We observe that Λ is larger when the hierarchies among the quartic couplings are smaller. Let us explain this feature by comparing the points C and L in the Fig. 2 as an example. The quartic couplings at the points for $\tan\beta = 1.0$ are calculated as

$$\begin{aligned} C(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) &\simeq (0.37, 0.37, 3.67, -1.54, -2.50), \quad \Lambda \simeq 3.95 \text{ TeV}, \\ L(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) &\simeq (1.48, 1.48, 2.56, 1.99, -2.04), \quad \Lambda \simeq 6.41 \text{ TeV}. \end{aligned}$$

²The points listed in Table 2 and 3 are the case with no custodial symmetry in the Higgs potential. We shall discuss the case with the symmetry separately.

From these values we obtain

$$\begin{aligned} 1.2 \leq \delta\lambda_{ij} &\equiv ||\lambda_i| - |\lambda_j|| \leq 3.3 \quad \text{for } C, \\ 0.52 \leq \delta\lambda_{ij} &\equiv ||\lambda_i| - |\lambda_j|| \leq 1.08 \quad \text{for } L, \end{aligned}$$

where $i (> j)$ runs from 1 to 5. The hierarchies among the quartic couplings become smaller and smaller to give larger cutoff as shown Table 2 when we move the points along $C \rightarrow G \rightarrow J \rightarrow L \rightarrow N$. $\delta\lambda_{ij}$ at the point N takes almost the same values with the point L. But the absolute values of each quartic coupling at N are somewhat larger than those of L, so that Λ at L becomes slightly larger than Λ at N. The same tendency for degenerate quartic couplings are also seen in Fig. 2 by $D \rightarrow E \rightarrow F \rightarrow G$ and $A \rightarrow D \rightarrow H \rightarrow K \rightarrow N$. These behaviors for smaller hierarchies among the couplings are common in three reference points we choose.

The maximum value of the cutoff in Table 1, 2 and 3 is obtained at the point G, L and K, respectively. The measure of the hierarchies among the quartic couplings defined by $\delta\bar{\lambda} \equiv \delta\lambda_{ij}^{\max} - \delta\lambda_{ij}^{\min}$ takes the smallest values at these points compared with other points in each reference point. In particular the masses of new particles are almost degenerate at the point L and K. $\delta\bar{\lambda}$ for L and K are about 0.56 and 0.74, respectively, but that for G is more than 2. This is because at the reference point I the parameter T is negative at the 1σ errors, and it is possible when a relation $M_{h(A)} < M_{H^\pm} < M_{A(h)}$ is satisfied, which means that there are still hierarchies among the quartic couplings. So Λ at G is not so large compared with Λ 's at each point of L and K.

Let us study the behaviors of the cutoff when the Higgs potential has the custodial symmetry. At the points A, D, H, K, N (A, D, G, J, N) in the Fig. 2 (3) the custodial symmetry, implying $M_{H^\pm} = M_h$ with $\tan\beta = 1$, exists in the Higgs potential. The other custodial symmetry realized by $M_{H^\pm} = M_A$ with $\tan\beta$ being free are obtained by exchanging the values between M_h and M_A at each point. The results for both cases are summarized in Tables 4 and 5 for the reference points II and III, respectively. We find, again,

that Λ takes larger value as the hierarchies among the quartic couplings are smaller; $A \rightarrow D \rightarrow H \rightarrow K \rightarrow N$ ($A \rightarrow D \rightarrow G \rightarrow J \rightarrow N$). At the last points N 's in the sequences $\delta \bar{\lambda}$ takes the smallest values compared with other points.

Now we shall discuss briefly the value of the cutoff when the mass of M_H becomes small. We vary the Higgs mass M_H to see how the values of Λ change and its sensitivity to the top quark. We analyze them at $M_H = 80, 60, 40, 20$ GeV for each value of $M_t = 175$ GeV and 125 GeV. From the previous analyses for the cutoff we found that Λ takes larger value when the masses of new particles are almost degenerate. So we assume $M_{H^\pm} = M_h = M_A$ ($\equiv M_{\text{new}}$) with $\tan \beta = 1.0$, which means $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_4 = -\lambda_5$ as mentioned in the section 3. We present results of Λ for these cases in Table 6. When M_H becomes light, the quartic couplings become smaller for both $M_t = 175$ GeV and $M_t = 125$ GeV cases to yield larger cutoff. In particular, for $M_t = 125$ GeV the degrees of decreasing the values of the quartic couplings are large compared with $M_t = 175$ GeV case, so that the difference of the cutoff between $M_t = 175$ GeV and $M_t = 125$ GeV for each value of M_H is enhanced as M_H becomes lighter. Apparently, we see the effect of the top quark on Λ . If we take $M_H = 10$ GeV for $M_t = 125$ GeV, which corresponds to

$$\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_4 = -\lambda_5 \simeq 0.45, \quad M_{\text{new}} \simeq 165 \text{ GeV},$$

Λ is about 1.06×10^{14} GeV. A large gauge hierarchy can be realized, but the constraints from the LEP bound of $M_H \simeq 64$ GeV and the announced heavy top quark mass are not satisfied. The top and Higgs masses are so heavy that the cutoff cannot reach the Planck or GUT scale.

5 Concluding Remarks

We have studied the one-loop RGE's in the electroweak theory with two massless Higgs doublets. Initial conditions of the equations for the quartic couplings are fixed by the constraints obtained from the oblique parameter T at three reference points $(M_t, M_H) = (175, 100), (150, 100), (125, 100)$ GeV. The constrained quartic couplings are large enough that the true minimum in the potential of the model is not destabilized by the heavy top quark. We

have found the cutoff Λ in the model for the reference points. The result is summarized as $\Lambda \simeq 0.52 \sim 8.4$ TeV. The values of the cutoff among these reference points are not so different. The cutoff becomes larger when the quartic couplings are almost degenerate, which also includes the case with the custodial symmetry in the Higgs potential. The value of the cutoff is at most about 60 TeV, and it is impossible to obtain larger values of Λ than this value from the current experimental limits on the Higgs and the top quark masses. The heavy top quark and LEP bound on the Higgs mass prevent Λ from reaching the high energy scale. It is impossible to realize the large gauge hierarchy as suggested many years ago by S. Weinberg in the electroweak theory with two massless Higgs doublets because of the constraints on Λ from the oblique parameter T and the heavy top and Higgs masses.

The obtained cutoff Λ indicates that the physics we have considered must be changed around Λ . It is natural to take a point of view that our model is an effective theory at low energy of some fundamental dynamics. When the energy scale approaches to Λ , the quartic couplings become large and are outside the validity of the perturbation theory. This implies that nonperturbative dynamics governs the physics near the scale Λ . It may be possible to consider that the massless Higgs fields are Nambu-Goldstone-boson fields associated with the breakdown of some symmetries. This possibility is natural in the usual sense that effective theories forbid scalar mass terms for naturalness. One may also consider that the Higgs potential (2.1) is regarded as a part of the low-energy effective (Ginsburg-Landau) lagrangian of composite Higgs fields, which are bound states of fermion and anti-fermion. In this case the mass terms for the composite Higgs fields at the tree level are renormalized to be zero, and one should take into account the effects of higher dimensional operators in discussing the low-energy physics because of a small hierarchy between $v \sim 246$ GeV and Λ we have studied.

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Figure Captions

Fig.1 The allowed mass regions for M_{H^\pm} and M_h from the oblique parameter T at the reference point I. Points denoted by A~H in the region give initial conditions of the quartic couplings for solving the RGE's. The quartic couplings in the right domain side of the allowed regions are calculable as explained in the text.

Fig.2 The allowed mass region for M_{H^\pm} and M_h from the oblique parameter T at the reference point II. Points denoted by A~Q in the region give initial conditions of the quartic couplings for solving the RGE's. At the points A, D, H, K and N the Higgs potential has the custodial symmetry which implies $M_{H^\pm} = M_h$ with $\tan \beta = 1.0$.

Fig.3 The allowed mass region for M_{H^\pm} and M_h from the oblique parameter T at the reference point III. Points denoted by A~Q in the region give initial conditions of the quartic couplings for solving the RGE's. At the points A, D, G, J and N the Higgs potential has the custodial symmetry which implies $M_{H^\pm} = M_h$ with $\tan \beta = 1.0$.

Table Captions

Table 1 The cutoff Λ obtained at the reference point $(M_t, M_H) = (175, 100)$ GeV for $\tan \beta = 0.6, 1.0, 3.0$. The value of M_A is determined once we fix the point on $M_{H^\pm} - M_h$ plane by the relation (3.2). The values of left (right) side in the row of Λ correspond to results obtained by quartic couplings in the left (right) domain of the allowed regions in Fig. 1.

Table 2 The cutoff Λ obtained at the reference point II for $\tan \beta = 0.6, 1.0, 3.0$. The value of M_A is determined once we fix the point on $M_{H^\pm} - M_h$ plane by the relation (3.2).

Table 3 The cutoff Λ obtained at the reference point III for $\tan \beta = 0.6, 1.0, 3.0$. The value of M_A is determined once we fix the point on $M_{H^\pm} - M_h$ plane by the relation (3.2).

Table 4 The cutoff Λ in the case with the custodial symmetry in the Higgs potential at the reference point II. The upper (lower) values in each point correspond to $M_{H^\pm} = M_h$ with $\tan \beta = 1.0$ ($M_{H^\pm} = M_A$ with $\tan \beta$ being free). GeV unit is used for the masses of new particles. $M_{i=j}$ means $M_i = M_j$.

Table 5 The cutoff Λ in the case with the custodial symmetry in the Higgs potential at the reference point III. The upper (lower) values in each point

correspond to $M_{H^\pm} = M_h$ with $\tan\beta = 1.0$ ($M_{H^\pm} = M_A$ with $\tan\beta$ being free). GeV unit is used for new particles. $M_{i=j}$ means $M_i = M_j$.

Table 6 The cutoff Λ for $M_H = 80, 60, 40, 20$ GeV at each value of $M_t = 175$ GeV and 125 GeV. The values in parentheses correspond to the case for $M_t = 125$ GeV. $\lambda_{1=2=3}$ and $\lambda_{4=5}$ mean $\lambda_1 = \lambda_2 = \lambda_3$ and $\lambda_4 = \lambda_5$, respectively.

Table 1

point	(M_h, M_A) (GeV)	$\tan \beta$	Λ (TeV)
A	(150, 484)	0.6	0.52 ; (0.115)
		1.0	0.65 ; 0.60
		3.0	0.55 ; (0.057)
B	(150, 474)	0.6	0.61 ; (0.121)
		1.0	0.79 ; 0.72
		3.0	0.63 ; (0.058)
C	(150, 453)	0.6	0.83 ; (0.134)
		1.0	1.14 ; 1.02
		3.0	0.77 ; (0.061)
D	(150, 410)	0.6	1.65 ; (0.168)
		1.0	2.45 ; 2.17
		3.0	0.95 ; (0.067)
E	(200, 471)	0.6	0.61 ; (0.122)
		1.0	0.83 ; 0.76
		3.0	(0.32) ; (0.059)
F	(200, 449)	0.6	0.83 ; (0.136)
		1.0	1.22 ; 1.10
		3.0	(0.33) ; (0.061)
G	(200, 406)	0.6	1.57 ; (0.177)
		1.0	2.71 ; 2.40
		3.0	(0.33) ; (0.068)
H	(240, 431)	0.6	0.97 ; (0.151)
		1.0	1.78 ; 1.57
		3.0	(0.18) ; (0.064)

Table 2

point	(M_h, M_A) (GeV)	$\tan \beta$	Λ (TeV)
B	(150, 472)	0.6	0.65
		1.0	0.81
		3.0	0.64
C	(150, 389)	0.6	2.52
		1.0	3.95
		3.0	1.06
E	(200, 458)	0.6	0.78
		1.0	1.09
		3.0	(0.328)
F	(200, 435)	0.6	1.12
		1.0	1.75
		3.0	(0.334)
G	(200, 385)	0.6	2.14
		1.0	4.4
		3.0	(0.338)
I	(250, 427)	0.6	0.99
		1.0	2.08
		3.0	(0.162)
J	(250, 374)	0.6	1.22
		1.0	5.25
		3.0	(0.162)
L	(300, 352)	0.6	0.55
		1.0	6.41
		3.0	(0.106)
M	(352, 300)	0.6	(0.29)
		1.0	5.93
		3.0	(0.080)
O	(427, 250)	0.6	(0.16)
		1.0	1.75
		3.0	(0.064)
P	(458, 200)	0.6	(0.13)
		1.0	0.97
		3.0	(0.061)
Q	(472, 150)	0.6	(0.12)
		1.0	0.74
		3.0	(0.058)

Table 3

point	(M_h, M_A) (GeV)	$\tan \beta$	Λ (TeV)
B	(150, 357)	0.6	3.55
		1.0	6.02
		3.0	1.14
C	(200, 466)	0.6	0.73
		1.0	0.95
		3.0	(0.328)
E	(200, 351)	0.6	2.56
		1.0	6.48
		3.0	(0.343)
F	(250, 456)	0.6	0.77
		1.0	1.19
		3.0	(0.162)
H	(250, 337)	0.6	1.29
		1.0	7.12
		3.0	(0.155)
I	(300, 431)	0.6	0.55
		1.0	2.09
		3.0	(0.105)
K	(300, 333)	0.6	0.59
		1.0	8.40
		3.0	(0.106)
L	(350, 407)	0.6	(0.310)
		1.0	3.44
		3.0	(0.081)
M	(350, 372)	0.6	(0.313)
		1.0	6.61
		3.0	(0.081)
O	(400, 361)	0.6	(0.199)
		1.0	3.20
		3.0	(0.068)
P	(400, 304)	0.6	(0.199)
		1.0	3.46
		3.0	(0.068)
Q	(440, 297)	0.6	(0.152)
		1.0	1.45
		3.0	(0.062)

Table 4

point	$(M_{H^\pm=h}, M_A)$ $(M_{H^\pm=A}, M_h)$	$\tan \beta$	Λ (TeV)
A	(150, 477)	1.0	0.73
		0.6	(0.121)
	(150, 477)	1.0	0.67
		3.0	(0.058)
D	(200, 469)	1.0	0.86
		0.6	(0.126)
	(200, 469)	1.0	2.44
		3.0	(0.059)
H	(250, 452)	1.0	1.24
		0.6	(0.138)
	(250, 452)	1.0	1.09
		3.0	(0.061)
K	(300, 413)	1.0	2.86
		0.6	(0.138)
	(300, 413)	1.0	2.35
		3.0	(0.061)
N	(350, 303)	1.0	6.03
		0.6	0.53
	(350, 303)	1.0	6.47
		3.0	(0.103)

Table 5

point	$(M_{H^\pm=h}, M_A)$ $(M_{H^\pm=A}, M_h)$	$\tan \beta$	Λ (TeV)
A	(150, 470)	1.0	0.86
		0.6	(0.113)
	(150, 470)	1.0	0.78
		3.0	(0.058)
D	(200, 462)	1.0	1.04
		0.6	(0.135)
	(200, 462)	1.0	0.94
		3.0	(0.059)
G	(250, 443)	1.0	1.58
		0.6	(0.149)
	(250, 443)	1.0	1.38
		3.0	(0.062)
J	(300, 402)	1.0	4.19
		0.6	(0.197)
	(300, 402)	1.0	3.34
		3.0	(0.068)
N	(350, 270)	1.0	7.07
		0.6	0.93
	(350, 270)	1.0	7.83
		3.0	(0.133)

Table 6

M_H (GeV)	M_{new} (GeV)	$\lambda_{1=2=3} = -\lambda_{4=5}$	Λ (GeV)
80	319 (301)	1.68 (1.50)	1.45×10^4 (3.17×10^4)
60	289 (264)	1.38 (1.15)	6.45×10^4 (2.74×10^5)
40	260 (224)	1.12 (0.83)	5.19×10^5 (1.80×10^7)
20	237 (182)	0.93 (0.54)	5.02×10^6 (1.50×10^{11})

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